Complex Cytochrome P450 kinetics due to multisubstrate binding and sequential metabolism. Part 1. Theoretical considerations.

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Running Title:

Complex CYP kinetics: Theoretical Considerations

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Number of text pages: 28
Number of tables: 1
Number of figures: 10
Number of references: 62
Number of words in
  Abstract: 139
  Introduction: 825
  Discussion: 1641

ABBREVIATIONS:

AICc: corrected Akaike information criterion, CL\textsubscript{int}: intrinsic clearance, CYP: cytochrome P450, DDI: drug-drug interaction, E: enzyme, EH: Eadie-Hofstee, ES: enzyme-substrate complex, ESS: enzyme-substrate-substrate complex, EP: enzyme-product complex, ES*: active oxygenating species derived from ES, ESS*: active oxygenating species derived from ESS, E\textsubscript{t}: total enzyme concentration, k: rate constants, k\textsubscript{cat}: catalytic turnover rate, K\textsubscript{m}: Michaelis-Menten
constant, ODE: ordinary differential equations, P: product, S: substrate, Seq: sequential, \( V_{\text{max}} \): maximal velocity

**Keywords**

Numerical methods, Michaelis-Menten, sequential metabolism, multiple substrate binding, atypical kinetics, cytochrome P450
Abstract

Complexities in CYP mediated metabolism kinetics include multisubstrate binding, multiple product formation and sequential metabolism. Saturation curves and intrinsic clearances were simulated for single substrate and multisubstrate models using derived velocity equations and numerical solutions of ordinary differential equations (ODEs). Multisubstrate models focused on sigmoidal kinetics due to their dramatic impact on clearance predictions. These models were combined with multiple product formation and sequential metabolism and simulations were performed with random error. Use of single substrate models to characterize multisubstrate data can result in inaccurate kinetic parameters and poor clearance predictions. Comparing results for use of standard velocity equations with ODEs clearly shows that ODEs are more versatile and provide better parameter estimates. It would be difficult derive concentration-velocity relationships for complex models, but these relationships can be easily modeled using numerical methods and ODEs.
Significance Statement

The impact of multisubstrate binding, multiple product formation, and sequential metabolism on the CYP kinetics was investigated. Numerical methods are capable of characterizing complicated CYP kinetics.
Introduction

Drug metabolism plays an important role in determining the pharmacokinetic and pharmacodynamic properties of drug candidates. Cytochrome P450s (CYPs) are a superfamily of enzymes involved in the metabolism of over 70% of drugs currently on the market (Zanger and Schwab, 2013). CYP-mediated clearance and related drug interactions can be a major issue during drug development. CYPs demonstrate unusual kinetics with respect to ligand selectivity (Ekroos and Sjögren, 2006) and multiple-substrate binding. In many cases, multi-substrate binding results in atypical saturation kinetics, including sigmoidal saturation, substrate inhibition, and biphasic saturation curves (Korzekwa et al., 1998; Tracy, 2006). Spectral binding and X-ray crystallography studies support the theory that the active sites of some CYPs are large and flexible (Shou et al., 1994; Ueng et al., 1997; Korzekwa et al., 1998; Hosea et al., 2000; Domanski et al., 2001; Ekins et al., 2003; Yoon et al., 2004) and can accommodate the simultaneous binding of multiple substrates (Li and Poulos, 2004; Wester et al., 2005; Roberts et al., 2011; Nguyen et al., 2016; Sevrioukova and Poulos, 2017). In addition to the simultaneous multiple binding of molecules of the same substrate, binding of different substrates occurs as well, resulting in heterotropic activation and inhibition (Ueng et al., 1997; Kenworthy et al., 2001; Hutzler and Tracy, 2002; Galetin et al., 2003; Collom et al., 2008; Blobaum et al., 2013). There are many possible factors that may be involved in non-Michaelis-Menten CYP kinetics, including active site flexibility, distinct binding sites (Hosea et al., 2000), and protein-protein interactions (Jamakhandi et al., 2007; Davydov et al., 2017; Dangi et al., 2021). However, most experimental saturation curves can be represented adequately by more simple ES and ESS models.
The combination of active site flexibility, fast rotation speed of substrate molecules in the active site, and versatile active oxygen species (Guengerich, 2018), results in multiple metabolite formation in a parallel or sequential manner (Masubuchi et al., 1996; Galetin et al., 2004; Obach, 2013). Multiple primary metabolites formed from the enzyme-substrate (ES) and enzyme-substrate-substrate (ESS) complex (Jones and Korzekwa, 1996) and secondary metabolite formation (Pang, 1995) add more complexity to CYP kinetics. Velocity equations as a function of substrate concentration and initial conditions are not easily derived for these complex schemes.

Since Cleland published methods for the least-squares analysis of enzyme kinetic data in 1979 (Cleland, 1979), there has been a steady movement away from graphical methods for kinetic analyses to robust statistical analyses by model fitting to derived velocity equations (e.g., the Michaelis-Menten equation) and various methods of numerical analysis (Hemker, 1972; Plant, 1979; Frenzen and Maini, 1988; Johnson, 2009; Kuzmič, 2009; Manimozhi et al., 2010; Rajendran et al., 2018; Yadav et al., 2021). For enzyme kinetic models with one independent variable (time) kinetic schemes can be represented as a collection of ordinary differential equations (ODEs). In addition to the derivation and use of velocity equations to model enzyme kinetics, ODEs can be used with numerical methods to directly model and parameterize complex kinetic schemes. The complex CYP kinetics encountered in time-dependent inactivation including inactivator binding, inhibitor depletion and sequential metabolism has been previously modeled using numerical analysis (Korzekwa et al., 2014; Nagar et al., 2014; Yadav et al., 2018; Yadav et al., 2020; Yadav et al., 2021).
CYP kinetics parameters are often used to determine the enzymes responsible for metabolite formation and to predict the potential for drug-drug interactions. Another commonly used CYP assay measures substrate depletion over time to estimate the intrinsic clearance (CLint) of a reaction. This is the first order rate constant observed at low substrate concentrations for most kinetic schemes. This clearance is then used to scale up to predict drug clearance in humans. It has been demonstrated that use of a different concentration range may lead to variable in vitro CLint estimations when sigmoidal kinetics are observed (Komura et al., 2005; Iwaki et al., 2019). Simulations have shown that Vmax/Km will not be accurate when sigmoidal kinetics is observed and the turnover rate from the ES complex approaches zero (Korzekwa, 2021).

With respect to sequential metabolism and multiple metabolites, numerical solutions have been reported previously (Frenzen and Maini, 1988; Varón et al., 2005), but no derived velocity equations have been reported, possibly due to the complex branched pathways that must be considered. The numerical method is facile for modeling multiple metabolite formation. Modeling metabolite exposure is a crucial issue in drug discovery and development especially for active (Obach, 2013) and toxic circulating metabolites (Schadt et al., 2018). Numerical analysis using ODEs is expected to easily provide accurate parameters for multiple metabolites, possibly leading to more accurate in vivo predictions of drug metabolism and metabolite disposition.

In Part 1 of these manuscripts, we demonstrate the use of the numerical analyses in characterizing Km, Vmax, and concentration-dependent CLint upon multi-substrate binding.
describing concentration-dependent metabolite ratios, and investigating sequential metabolism. In Part 2, in-house data of three different model drugs - midazolam, ticlopidine and diazepam were generated and analyzed with this modeling strategy.
**Materials and Methods**

**Theoretical Considerations.** Fig. 1A shows the traditional single-substrate binding model, in which the EP complex is assumed to be short-lived compared to the ES complex. The equation derived from this scheme is the well-known Michaelis-Menten equation (Michaelis and Menten, 1913).

\[
\frac{\text{velocity} (v)}{E_t} = \frac{v_{\text{max}} [S]}{K_m + [S]} = \frac{k_{\text{cat}} [S]}{K_m + [S]}
\]  

(1)

Where \( k_{\text{cat}} \) is the maximum velocity at unit enzyme concentration \( (E_t) \), \([S]\) is the substrate concentration, and \( K_m \) is the substrate concentration at half-maximum turnover rate. Fig. 1B shows a two-substrate model, which can result in non-Michaelis-Menten kinetics. Evidence of non-Michaelis-Menten (atypical) kinetics has been reported widely for many CYPs, reactions, and substrates (Atkins, 2005; Atkins, 2006). Multiple substrate binding in the enzyme active site has been confirmed with X-ray crystallography (Yano et al., 2004; Gay et al., 2010). Most categories of atypical kinetics can be explained by Fig. 1B, where the specific binding orientation of substrates is not assumed. For these models, the first substrate binds to the enzyme active site with an apparent \( K_{m1} = (k_2+k_3)/k_1 \) and turnover rate \( k_{\text{cat1}} = k_3 \). A second substrate binds to the enzyme active site with an affinity \( K_{m2} = (k_5+k_6)/k_4 \) and turnover rate \( k_{\text{cat2}} = k_6 \) (Fig. 1C). The \( K_m \) values are apparent, since both processes occur simultaneously. Sigmoidal kinetics can be observed when \( K_{m1} > K_{m2} \) or \( k_{\text{cat1}} < k_{\text{cat2}} \). Biphasic kinetics is observed when \( K_{m1} < K_{m2} \) and \( k_{\text{cat1}} < k_{\text{cat2}} \). Substrate inhibition occurs at the condition of \( K_{m1} < K_{m2} \) and \( k_{\text{cat1}} > k_{\text{cat2}} \).

Due to large and flexible binding pockets and non-specific substrate orientation for some CYPs (Shou et al., 1994; Ueng et al., 1997; Korzekwa et al., 1998; Hosea et al., 2000; Domanski et al., 2001; Ekins et al., 2003; Yoon et al., 2004), formation of multiple metabolites by a single CYP is
common (Masubuchi et al., 1996; Galetin et al., 2004; Obach, 2013). Kinetic schemes that include the formation of two metabolites are shown in Fig. 2. The schemes in Figs. 2A and B can correctly model the formation of two products when product formation is rate-limiting. However, for CYPs, oxygen activation is rate-limiting and substrate oxidation is fast (Luthra et al., 2011). The branch between formation of the two products occurs from the activated oxygen species and is not rate-limiting within the catalytic cycle (i.e., the branch is product determining but not rate-limiting). As described below, it is necessary to use the kinetic schemes in Figs. 2C and D to describe multi product CYP kinetics. In these schemes, ES* and ESS* represent the activated oxygen species. It should be noted that $K_{m1,mP1} = K_{m1,P2}$ and $K_{m2,P1} = K_{m2,P2}$ is assumed since both products are formed from the same enzyme-substrate complex.

**Dataset simulation.** Saturation curves (velocity versus substrate concentration) were simulated as follows: 1) constructing ordinary differential equations (ODEs) for the kinetic schemes in Fig.1 and 2; 2) varying either the dissociation or turnover rate constant; 3) simulating saturation curve data with or without adding random error; and 4) directly fitting the Michaelis-Menten equation, ESSP rate equation, and ODEs for the schemes in Fig.1 and 2 to the simulated dataset and comparing the fitted parameters with those used in the simulation. Simulations were repeated 500 times, and the results were checked to determine convergence and the distribution of parameter estimates.

All association rate constants (e.g., $k_1$ and $k_4$ in Fig. 1) were set to 270 $\mu$M$^{-1}$ min$^{-1}$ (Barnaba et al., 2016). Simulated dissociation rate constants were selected to achieve the $K_{m1}$ and $K_{m2}$ values in the range of 1-1000 $\mu$M (Walsky and Obach, 2004), and turnover rate constants ($k_{cat1}$ and $k_{cat2}$)
were set within the range of 0 - 60 min\(^{-1}\). The values were chosen to provide the desired saturation kinetic profiles (e.g., sigmoidal kinetics, [P2] > [P1], etc.). For simulation of saturation curves, initial substrate concentrations were varied between 0 and 300 µM, \(E_t\) was 5 nM and the incubation time were set at 5, 10, or 20 min. Substrate consumption in all saturation curve simulations was <10% unless indicated. CL\(_{\text{int}}\) was simulated as \(d[S]/dt / [S]\), over a range of 0.01 – 1 µM initial substrate concentrations, with \(E_t = 100\) nM. Triplicate data with random error of substrate concentrations and incubation time were simulated from ESSP and ESSP1P2 ODEs, respectively. For some simulations and parameterizations, correlation between parameters is expected, and some parameters must be held constant. For example, if an enzyme species is not saturated within the range of substrate concentrations, both \(k_{\text{cat}}\) and \(K_m\) cannot be determined. One parameter must be held constant and only the ratio of \(k_{\text{cat}}/K_m\) can be determined.

Simulated datasets were generated via the built-in function NDSolve in Mathematica 12.0 (Wolfram Research, Champaign, IL). Random errors were generated from RandomVariate and NormalDistribution function by setting the mean as one and relative standard deviation as 1%, 5%, 10%, and 20%. This error was multiplied by the simulated values to generate log-normally distributed values. For the simulations, 500 runs of triplicate data were generated at each error level.

**Model fitting.** For the derived velocity equation method, the Michaelis-Menten equation and ESSP rate equation (Eq. 2) were fit to the simulated concentration-velocity data. The NonlinearModelFit function was used to parameterize the model with 1/Y weighting.

\[
\frac{\text{velocity} \ (v)}{E_t} = \frac{k_{\text{cat}} S + \frac{k_{\text{cat}}}{K_m} S^2}{K_m + S + \frac{S^2}{K_m}} \quad (2)
\]
For numerical analyses, each metabolite concentration dataset (triplicate) was used to parameterize the micro rate constants of the ESP and ESSP ODEs using NDSolve (MaxSteps → 10,000 and PrecisionGoal → ∞) and NonlinearModelFit with 1/Y weighting. Two metabolite concentration datasets (triplicate) were used to simultaneously parameterize the micro rate constants of the ESP1P2 and ESSP1P2 ODEs.
Results

Two-substrate binding and single product formation. Two substrate binding can lead to atypical saturation kinetic behavior (biphasic, substrate inhibition and sigmoidal). Sigmoidal kinetics can be observed for two general scenarios: \( K_{m2} < K_{m1} \) and/or \( k_{cat2} > k_{cat1} \). These general cases and the specific cases when \( k_{cat1} = 0 \) are listed in Table 1 and simulations are shown in Fig. 3. Fig. 3 shows simulations for four possible scenarios of sigmoidal kinetics. All of the plots in Fig. 3 show sigmoidal saturation. The impact of relative differences in kinetic parameters on the degree of sigmoidicity is also shown. When \( k_3 > 0 \), a linear range is observed at low concentrations (Figs. 3B and 3C). When \( k_3 = 0 \), no linear range is observed (Figs. 3C and 3D). The implication of not having a linear range is significant. In the field of drug metabolism, a linear \( V_{max}/K_m \) describes a constant in vitro intrinsic clearance. Fig. 4 shows the concentration-dependence of \( CL_{int} \) for the sigmoidal kinetic curves in Fig. 3. In Cases I and II, at most a 2-fold difference in \( CL_{int} \) would be observed at concentrations < 1 µM. It is noteworthy that up to a 100-fold difference in \( CL_{int} \) may be observed in Cases III and IV.

Two-substrate binding and two product formation. Although CYPs can usually form multiple metabolites, most of the kinetic models for atypical kinetics consider the dominant metabolite. Although some CYPs can form more then two primary metabolites, for simplicity we have simulated the formation of two metabolites from the same CYP. The scheme in Fig. 5A assumes that the formation steps for both products are rate-limiting and P1 shows sigmoidal kinetics (Fig. 5B) since ESS formation has a higher velocity for P1 formation. Although P2 appears to have an almost hyperbolic saturation curve (Fig. 5C), the Eadie Hofstee (EH) plot shows the kinetics are atypical. This is because there is some minimal sigmoidicity due to ESS formation, even when
the apparent $K_m$ and $k_{cat}$ for ES and ESS values are identical. Similar to Fig. 4, the higher the $k_7/k_3$ ratio, the greater the change in P1 formation CL$_{int}$ with increasing [S]. P2 formation CL$_{int}$ is constant when varying $k_7$, since substrate oxidation is not rate-limiting.

The scheme in Fig. 6A assumes that formation of active oxygenating species is rate-limiting. Again, P1 shows sigmoidal kinetics (Fig. 6B), but P2 now shows substrate inhibition (Fig. 6C). Substrate inhibition increases for P2 with increased $k_7/k_3$ ratio.

The probability distribution of parameters to compare the use of single versus multiple substrate models, velocity equations versus ODEs, and single versus multiple product formation is shown in Fig. 7. These simulations ($n = 500$) were performed using a multiple substrate, multiple product scheme (ESSP1P2) at error levels of 1%, 5% and 10%. In all cases, ESS models were necessary to accurately parameterize $K_{m1}$ and $k_{cat1}$. Although the means were approximately the same for all ESS models, the distribution was narrowest for the ESS models when both products were modeled simultaneously. Parameters for all models tested are listed in Supplementary Table S1.

Sequential metabolism. CYPs can also metabolize substrates in sequentially to multiple metabolites. In this section we simulated sequential metabolism using single-substrate (Fig. 8) and two-substrate binding kinetic models (Fig. 9). The kinetic scheme in Fig. 8A includes the EP1 intermediate. If P2 formation from P1 is slower than the P1 release rate, P2 will show a lag time since accumulation and rebinding of P1 to the CYP active site is necessary for P2 formation (Fig. 8B). If P2 formation from P1 is much faster than the P1 release rate, P2 will not show a lag
time. Assuming linear product formation for P1, the velocities measured at different incubation time yields the same saturation curves for P1 but not with P2 (Fig. 8C).

Fig. 8D shows the relationship of the rate of P1 release ($k_5$) and the sequential metabolism turnover ($k_6$). At $k_6 = 27 \text{ min}^{-1}$, varying the ratio of $k_5/k_6$ from 100 to 1 results in P1 dissociation constants from 10 to $0.1 \mu \text{M}$. Increasing the rate of P2 formation leads to a decrease in P1 formation and an increase in P2. Substrate inhibition for P2 formation becomes apparent in the Eadie-Hofstee plot since substrate competes for free enzyme with the rebinding of P1.

Sequential metabolism was also simulated with a two-substrate binding kinetic model (Fig. 9A). However, this model assumes P1 release is fast and must compete with substrate binding for free enzyme. Provided that P1 release is faster than P2 formation, these simulations should be relevant. Three types of ESS formation were simulated: when $k_3 = k_6$, $k_3 > k_6$ and at $k_3 < k_6$ (Figs. 9B-D, respectively). An increase in P1 affinity led to a decrease in velocity of P1 and increase in the formation of P2. As expected, P1 formation shows slight sigmoidal, substrate inhibition, and sigmoidal kinetics for $k_3 = k_6$, $k_3 > k_6$ and at $k_3 < k_6$, respectively. The saturation curve for P2 follows that for P1 at low [S]. At high [S], P2 formation shows substrate inhibition for all cases since increasing substrate concentrations competes with P1 rebinding.

Using numerical methods, any of the above approaches to model multi-product and sequential metabolism can be combined. Fig. 10A shows a scheme where two products (P1 and P2) can be formed from an ESS model, and one of the products is further metabolized to a third product, P3. In Fig. 10B, both initial products can be metabolized to the same secondary product by
sequential metabolism. For the simulations in Fig. 10C, the substrate binding constants were held constant with $k_1, k_5$ and $k_9 = 270$ μM$^{-1}$ min$^{-1}$, $k_2$ and $k_6 = 10000$ min$^{-1}$ (Rittle and Green, 2010), and the turnover rates for P1 and P2 formation were held constant with $k_3 = 1$ min$^{-1}$, $k_7 = 100$ min$^{-1}$, and $k_4 = k_8 = 1$ min$^{-1}$. In these simulations, the dissociation constants for P1 binding were modeled at 10 and 0.1 μM. In Fig 10C we can see that as the affinity for P1 to the enzyme increases, P1 is rapidly converted to P3 and the ratio of P1 to P2 is decreased. As substrate concentrations are increased, P1 binding is inhibited and the expected change in P1/P2 ratios are observed. Finally, Fig. 10D shows that both the affinity and rates of sequential metabolism affect the P1/P2 ratios at different substrate concentrations.
Discussion

The goals of this manuscript (Part 1) are four-fold: 1) to simulate and evaluate atypical kinetics using both derived velocity equations and numerical solutions using ODEs and compare the results; 2) to evaluate the impact of multiple metabolites on these models; 3) to evaluate the impact of atypical kinetics for reactions that show sequential metabolism; and 4) to provide the theoretical basis for analysis of datasets discussed in Part 2 of these manuscripts. The focus of this manuscript is on schemes and parameters that display sigmoidal saturation kinetics since these results are the most difficult to interpret.

Two standard in vitro experiments to characterize CYP-mediated oxidations are saturation curves and substrate depletion assays. Substrate depletion assays observe the loss of substrate over time, usually in a microsomal incubation. This assay only requires quantitation of the substrate, a simple task since the structure is known. The assay provides an initial estimate of hepatic stability. If first-order elimination is assumed, the elimination rate constant can be used to estimate intrinsic clearance (CL\text{int}). This clearance value can be scaled up to a human CL\text{int} using standard methods (Ito et al., 1998; Chiba et al., 2009). Once metabolites are known and standards are available, saturation curves are used to determine kinetic constants i.e. substrate affinities and velocities. Comparing parameters for different enzymes, one can determine which CYPs form which metabolites and which enzymes are likely to be important clinically. These kinetic parameters can also be used to predict whether a drug will be a victim or perpetrator of drug interactions.
Non-Michaelis-Menten kinetics are often observed for CYP-mediated oxidation reactions, resulting in non-hyperbolic saturation curves, for example, sigmoidal saturation, substrate inhibition, and biphasic saturation curves (Korzekwa et al., 1998). These saturation curves are a result of multiple substrates binding simultaneously to the enzyme (e.g. Fig. 1B). Most CYP-catalyzed reactions saturate well above unbound therapeutic concentrations. This results in linear pharmacokinetics since drug elimination occurs in the linear portion of the saturation curve. For substrate inhibition and biphasic saturation curves, the saturation curve is approximately linear below the $K_m$ for ES formation. This is because the binding of the first substrate (higher affinity binding) still has a linear region at low substrate concentrations. Substrate inhibition and biphasic saturation would have to occur at very low substrate concentrations to be clinically relevant. On the other hand, sigmoidal saturation curves can be more problematic. Sigmoidal kinetics can occur when either ESS formation occurs with a higher affinity than ES formation or if $k_{cat2}$ from ESS is faster than $k_{cat1}$ from ES (e.g. $k_6 > k_3$ in Fig. 1B). If $k_{cat1}$ contributes substantially at therapeutic concentrations, there may still be a linear region. However, if $k_{cat1}$ approaches zero, there may not be a linear region at low substrate concentrations. This will result in a different estimate of intrinsic clearance for different initial substrate concentrations (e.g. see Fig. 4). An example of the impact of using an Michaelis-Menten model to predict the intrinsic clearance of an enzyme showing sigmoidal kinetics is shown in Part 2 of these manuscripts.

The results in Fig. 3 are as expected, with the sigmoidal saturation curves having a linear region at low substrate concentrations when $k_{cat1}$ ($k_3$) is $> 0$ (Figs. 3B and C) and no linear region when $k_3 = 0$ (Figs. 3D and E). All simulations give the expected Eadie-Hofstee plots showing convex curvature to the right. Fig. 4 shows the impact of low $k_{cat1}$ values on CL$_{int}$. When $k_{cat1}$ is 1,
Sigmoidicity due to either higher affinity binding of the second substrate or higher velocity of the second substrate results in modest increases in $\text{CL}_{\text{int}}$ (~2 fold between 10 nM and 1 μM for 100-fold increases in affinity or velocity for ESS; Figs. 4A and B). However, when $k_{\text{cat}1} = 0$, ~100-fold increases in $\text{CL}_{\text{int}}$ are observed for $[S]$ between 10 nM and 1 μM (Figs. 4C and D). Also, there is no range where $\text{CL}_{\text{int}}$ is constant. An ~100-fold increase in $\text{CL}_{\text{int}}$ would be observed for $[S]$ between 0.1 nM and 10 nM as well. This nonlinearity can be identified with substrate depletion assays at different substrate concentrations. Predicting in vivo $\text{CL}_{\text{int}}$ would require conducting in vitro assays at $[S]$ that match the unbound intracellular concentrations. Also, concentration-dependent $\text{CL}_{\text{int}}$ in vitro may result in non-linear pharmacokinetics in vivo.

Sequential metabolism by the CYPs is important in drug discovery and development, since metabolites can be active, toxic, or cause drug-drug interactions (Jackson et al., 2018; Wienkers and Rock, 2021). We have simulated CYP kinetic models that can form two different products from either a single substrate complex (Figs. 2A and C) or from an ESS complex (Figs. 2B and D). These models do not include both multiple product formation and sequential metabolism. If a substrate can only bind once (Figs. 2A and C) the P1/P2 product ratio remains constant at $k_3/k_4$. However, for multi-substrate kinetics, metabolite ratios can change with substrate concentrations. Fig. 5 is the simplest scheme for two products formed from ES and ESS complexes. The simulations in Fig. 5 shows increasing formation of P1 from ESS while holding all other parameters constant. As expected, P1 formation shows increasing sigmoidicity as $k_7$ increases due to the higher velocity for P1 formation from ESS. However, this scheme does not accurately capture the impact of increasing one product formation pathway for a CYP reaction. As shown in Fig. 5C, changing $k_7$ has no impact on P2 formation. This is because binding is fast.
and the rate-limiting steps in this scheme are the product formation steps \((k_3, k_4, k_7, \text{ and } k_8)\). In reality, the rate-limiting step for CYP oxidations is the formation of an active oxygenating species, and this active oxygenating species can form different metabolites. This results in the CYP property called “metabolic switching” where blocking one site results in increased metabolism at other sites. This can be best modeled using the schemes in Figs. 2C and D and Fig. 6. In Fig. 6C, increasing \(k_7\) while holding all parameters constant results in a decrease in P2 formation at high substrate concentrations. Since formation of the active oxygenating species is rate-limiting, increasing P1 formation must result in a decrease in P2 formation. For a chemist modifying a molecule for stability, the difference between the schemes in Figs. 5 and 6 is important, since decreasing metabolism at one position may increase metabolism at another.

However, when fitting kinetic models to experimental datasets for multiple products, the simpler scheme in Fig. 5 is preferred. The same substrate inhibition profile for P2 seen in Fig. 6C would also be observed if \(k_8 < k_4\). Using the method of Cleland (Cleland, 1975) and the scheme in Fig. 6, net rate constants can be calculated for conversion of ES and ESS complexes to products, and the result is the simplified scheme in Fig. 5. For example, the net rate constant for P1 formation from ES in Fig. 6 \((k_{a1} k_3/(k_3+k_4))\) will be equal to \(k_3\) in Fig. 5.

The parameter probability distributions for the various simulations shown in Fig. 7 shows that 1) using an ES model for ESS kinetics results in inaccurate \(K_m\) and \(k_{cat}\) values; 2) use of ODEs instead of derived velocity equations minimizes the overall parameter errors; and 3) simultaneous fitting to both P1 and P2 data further decreases the parameter errors. These results suggest that using ODEs to model CYP kinetics is generally preferred to using derived velocity
equations. Derived velocity equations include additional simplifying assumptions, and equations for complex systems these equations are generally not available.

When sequential metabolism occurs, the rate of the first product release (P1 in Fig. 8A) has a large impact on the kinetics of the sequential product (P2). If P1 release is fast, there will be a lag time for P2 formation, since P1 must accumulate before P2 can be formed. Also, P2 formation can be inhibited by high substrate concentrations. Lag times have frequently been observed for time-dependent inhibition kinetics when multiple oxidation steps are required for CYP inactivation. Fig. 9 considers a scenario including multiple substrate binding and sequential metabolism with mandatory P1 release. A complete model would additionally include ESP1, ESP2, EP1P1, EP1P2 and EP2P2. Limiting schemes to mandatory product release may not always be sufficient to model experimental data (see Part 2). Finally, schemes can be readily constructed for multiple substrate binding with multiple product formation as well sequential metabolism (Fig. 10). Adding sequential metabolism can result in unusual P1/P2 product ratio curves when one or both products are further metabolized. A wide range of product ratio curves is possible for a variety of schemes and kinetic scenarios (e.g. see Fig. 10D). When experimental data are available for both P1 and P2 formation, product ratio plots can be very useful in selecting appropriate kinetic schemes (see Part 2).

Mechanisms can vary for different substrates with the same CYP, and for different CYPs with the same substrate. The diversity of possible mechanisms requires that statistical methods be used to determine the best model for a given dataset. We use AICc values to compare different mechanisms since this method can be used for nested and non-nested models. It is also important
to plot residuals. A run of signs indicates that the model is likely inadequate and may bias data interpretation. Parameter errors should be compared to the parameter estimates, since covariance will result in large relative parameter errors. This can also be observed in the correlation matrix for the calculation. It should be noted that insufficient data and high experimental errors can result in inaccurate model selection.

Finally, in Part 2 of these manuscripts, we have used numerical methods to simultaneously model the multiple metabolites formed initially and sequentially from diazepam by CYP3A4. This resulted in a mechanistic interpretation of diazepam metabolism that provided affinities and velocities consistent with the observed metabolic data for diazepam and its metabolites.

In conclusion, we have modeled combinations of multiple substrate binding, multiple product formation and sequential metabolism. We have focused on sigmoidal kinetics since it can have a dramatic impact on clearance predictions. The use of Michaelis-Menten models to characterize multi-substrate saturation data results in inaccurate kinetic parameters and clearance predictions. Comparing results for use of standard velocity equations with ODEs clearly shows that ODEs are more versatile and provide better parameter estimates. The complexity of the kinetic schemes used for these analyses shows that most kinetic schemes can be modeled, and these models can be parameterized, provided that sufficient experimental data is available. Finally, these analyses provide a framework for modeling the experimental CYP kinetics observed in Part 2 of these manuscripts.
Authorship Contributions

Participated in research design: Wang Z, Paragas EM, Korzekwa K

Conducted experiments: Wang Z, Paragas EM

Performed data analysis: Wang Z, Paragas EM, Nagar S, Korzekwa K

Wrote or contributed to the writing of the manuscript: Wang Z, Paragas EM, Nagar S, Korzekwa K
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Footnotes

This work was supported by the National Institutes of Health National Institute of General Medical Sciences (to K.K. and S.N.) [Grants 2R01GM104178 and 2R01GM114369].

Financial Disclosure and Conflict of Interest: The authors have no financial disclosures, and no conflicts of interest to report.
Figure Legends

Figure 1. Schemes for single product formation. (A) A single-substrate binding (ES complex), one metabolite formation (ESP) scheme is depicted. EP complex is not shown since fast product release is assumed. (B) A two-substrate binding (ESS complex), one metabolite formation (ESSP) scheme is depicted. EP, ESP, and EPP are not shown since fast product release and no interaction between substrate and product within the active site are assumed. (C) Expressions for $K_m$ and $k_{cat}$ for schemes A (dashed purple box) and B (dashed blue box) are listed. Rate constants for all reactions are denoted by $k_1$–$k_6$. The species depicted in the schemes are defined as follows: E, unbound enzyme; P1, product 1; ES, enzyme-substrate complex; and ESS, enzyme-substrate-substrate complex.

Figure 2. Schemes for multiple product formation. (A) A single-substrate binding, two metabolite formation (ESP1P2) scheme. (B) A two-substrate binding, two metabolite formation (ESSP1P2) scheme. (C) Single-substrate binding with rate-limiting oxygen activation. (D) Multiple-substrate binding with rate-limiting oxygen activation. (E) Expressions for $K_m$ and $k_{cat}$ for schemes A (dashed purple box) and B (dashed blue box) are listed. Rate constants for all reactions are denoted by $k_1$–$k_8$. Rate constants $k_{a1}$ and $k_{a2}$ denote oxygen activation steps for ES* and ESS* respectively. The species depicted in the schemes are defined as follows: E, unbound enzyme; P1, product 1; P2, product 2; ES, enzyme-substrate complex; and ESS, enzyme-substrate-substrate complex; ES* and ESS*, active oxygen species.
Figure 3. Sigmoidal saturation curves for multiple-substrate binding and single product formation. (A) Enzyme kinetic scheme. (B-E) Top: saturation curves (0-300 µM) with inset Eadie-Hofstee plots; Bottom: saturation curves (0-0.1 µM). Common micro rate constants: $k_1 = k_4 = 270 \text{ µM}^{-1}\text{min}^{-1}$, $k_2 = 22000 \text{ min}^{-1}$. Other fixed rate constants are shown at the top of each column and varying rate constants are at the bottom. Units for $k_3$, $k_5$, $k_6$ are $\text{min}^{-1}$.

Figure 4. Concentration dependence of $\text{CL}^\text{int}$ for the ESS model. (A-D) $\text{CL}^\text{int}$ as a function of $[S]$ for scenarios in Table 1 and Fig. 3. The arrows indicate the fold changes in $\text{CL}^\text{int}$ at 0.01, 0.1 and 1 µM. Common micro rate constants: $k_1 = k_4 = 270 \text{ µM}^{-1}\text{min}^{-1}$, $k_2 = 22000 \text{ min}^{-1}$. Units for $k_3$, $k_5$, $k_6$ are $\text{min}^{-1}$. Note the different scales of the Y-axes.

Figure 5. Saturation curves for multiple-substrate binding and multiple product formation. The scheme assumes that oxygen activation is not rate-limiting. (A) Enzyme kinetic scheme. (B-C) Top: saturation curves for P1 and P2 respectively (0-300 µM) with inset Eadie-Hofstee plots; Bottom: saturation curves for P1 and P2 respectively (0-0.1 µM). (D): $[\text{P1}]/[\text{P2}]$ metabolite ratio versus $[S]$, with inset at low $[S]$. The dashed lines represent the $k_3/k_4$ and $k_7/k_8$ ratios. Fixed micro rate constants: $k_1 = k_5 = 270 \text{ µM}^{-1}\text{min}^{-1}$, $k_2 = k_6 = 10000 \text{ min}^{-1}$, $k_3 = k_4 = k_7 = k_8 = 1 \text{ min}^{-1}$.

Figure 6. Saturation curves for multiple-substrate binding and multiple product formation. The scheme assumes that oxygen activation is rate-limiting. (A) Enzyme kinetic scheme. (B-C) Top: saturation curves for P1 and P2 respectively (0-300 µM) with inset Eadie-Hofstee plots; Bottom: saturation curves for P1 and P2 respectively (0-0.1 µM). (D): $[\text{P1}]/[\text{P2}]$ metabolite ratio versus $[S]$, with inset at low $[S]$. The dashed lines represent the $k_3/k_4$ and $k_7/k_8$ ratios. Fixed micro rate...
constants: $k_1 = k_5 = 270 \, \mu M^{-1} \, min^{-1}$, $k_2 = k_6 = 10000 \, min^{-1}$, $k_3 = k_4 = k_8 = 1000000 \, min^{-1}$, $k_{a1} = k_{a2} = 30 \, min^{-1}$.

Figure 7. The probability distribution of $K_{m1}$, $K_{m2}$, $k_{cat1}$, and $k_{cat2}$ estimates. (A) $K_{m1}$ at 1%, 5% and 10% error. (B) $k_{cat1}$ at 1%, 5% and 10% error. (C) $K_{m2}$ at 1%, 5% and 10% error. (D) $k_{cat2}$ at 1%, 5% and 10% error. The probability distribution for all parameters is depicted for the Michaelis-Menten equation (red), ESS velocity equation (blue), ES ODE (green), ESS ODE (purple), ESP1P2 ODE (orange), and ESSP1P2 ODE (magenta). Distribution is shown for 500 runs at each condition. Data were simulated with the following parameters: $k_1 = k_5 = 270 \, \mu M^{-1} \, min^{-1}$, $k_2 = 13500 \, min^{-1}$, $k_3 = 1 \, min^{-1}$, $k_4 = 1 \, min^{-1}$, $k_6 = 2700 \, min^{-1}$, $k_7 = 5 \, min^{-1}$, $k_8 = 1 \, min^{-1}$.

Figure 8. Single-substrate binding and sequential metabolism. (A) Enzyme kinetic scheme. (B) P2 concentration over time at $k_5/k_6 = 100$ versus 1, exhibiting a lag time in P2 formation at $k_5/k_6=100$. (C) Saturation curve plots for P1 and P2 simulated at different incubation time ($t$) at the following rate constants: $k_1 = 270 \, \mu M^{-1} \, min^{-1}$; $k_2 = 2700 \, min^{-1}$; $k_3 = 20 \, min^{-1}$; $k_4 = 270 \, \mu M^{-1} \, min^{-1}$; $k_5 = 2700 \,min^{-1}$; and $k_6 = 27 \, min^{-1}$. (D) Saturation curve plots for Products 1 and 2 simulated at different rates of product release ($k_5$) and sequential metabolism turnover ($k_6$). Insets: Eadie-Hofstee plots.

Figure 9. Multiple-substrate binding and sequential metabolism. (A) Enzyme kinetic scheme. Saturation curve plots for P1 and P2 simulated at different rates of product dissociation ($k_8$) and sequential metabolism turnover ($k_9$), when (B) $k_3 = k_6 = 27 \, min^{-1}$. (C) $k_3 > k_6$; $k_3 = 100 \,min^{-1}$ and $k_6 = 27 \,min^{-1}$, and (D) $k_3 < k_6$; $k_3 = 27 \,min^{-1}$ and $k_6 = 100 \,min^{-1}$. Fixed values used were: $k_1$, $k_4$.
and $k_7 = 270 \, \mu M^{-1} \, \text{min}^{-1}$, $k_2 = 22000 \, \text{min}^{-1}$, $k_5 = 11000 \, \text{min}^{-1}$, $k_9 = 27 \, \text{min}^{-1}$. All insets depict Eadie-Hofstee plots.

Figure 10. Models for multiple-substrate binding and multiple product formation where (A) one or (B) two products undergo sequential metabolism. (C) $[P1]/[P2]$ product ratio versus $[S]$ for model A simulated at increasing sequential turnover rate ($k_{11}$) with $k_{10}$ fixed at 2700 and 27 min$^{-1}$. (D) $[P1]/[P2]$ product ratio versus $[S]$ for model B simulated at increasing sequential turnover rate of P2 forming P3 ($k_{14}$) with $k_{10}$ fixed at 2700 and 27 min$^{-1}$. Common micro rate constants: $k_1$, $k_5$, and $k_9 = 270 \, \text{min}^{-1}$, $k_2$ and $k_6 = 10000 \, \mu M^{-1} \, \text{min}^{-1}$, $k_3 = 1 \, \text{min}^{-1}$ and $k_7 = 100 \, \text{min}^{-1}$, $k_4$ and $k_8 = 1 \, \text{min}^{-1}$. For D, $k_{12} = 270 \, \text{min}^{-1}$ and $k_{11} = 1 \, \text{min}^{-1}$.
### Tables

#### Table 1. Different cases for sigmoidal kinetics

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<tr>
<th>Parameter</th>
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</table>

The cases $k_2 > k_5$ ($K_{m1} > K_{m2}$) and $k_6 > k_3 = 0$ ($k_{cat1} > k_{cat2} > 0$) are not discussed.
Figure 1.
Figure 3.
Figure 4.
Figure 5.
Figure 6.
Figure 7.
Figure 8.
Figure 9.
Figure 10.